Research Statement

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In my research I have primarily focused on studying the mathematical structures and properties of low-order models (LOMs) for problems of atmospheric dynamics. The governing equations for these problems consist of systems of nonlinear partial differential equations (PDEs), making their analysis quite difficult. A common workaround, established in the pioneering works of Kolmogorov, Lorenz, and Obukhov, is to approximate these equations with finite systems of nonlinear ordinary differential equations (ODEs). These ODEs are commonly derived from the PDEs via the Galerkin method: the fluid dynamical fields are expanded into infinite series in time-independent basis functions (commonly Fourier modes), then the series are truncated and substituted into the PDEs yielding a finite system of ODEs (a LOM) for the time evolution of the coefficients in the truncated expansions. The most well-known such LOM is the celebrated Lorenz model of two-dimension Rayleigh-Bénard convection (2D RBC).

However, a common mistake that can happen with LOMs is when arbitrary truncations in the Galerkin method are used. This may lead to models that lack the fundamental physical properties of the original equations, such as energy conservation. For example, there are many instances in the literature of trying to extend the Lorenz model to larger, more realistic models of atmospheric dynamics, only to find that they lack energy conservation. A way to address this problem was to develop a class of LOMs with sound physical behavior, now called G-models. G-models consist of one or more coupled Volterra gyrostats, and their structure guarantees energy conservation in the frictionless limit. In fact, it has been demonstrated that the Lorenz model is equivalent to the simplest Volterra gyrostat with added constant forcing and linear friction, meaning that it is a G-model.

My research focuses on how G-models can be used to study various problems in atmospheric dynamics in a physically sound way. Using *Mathematica*, I developed an algorithm that permits the easy derivation of many LOMs, the study of their properties, and the determination of which of these LOMs has a G-model form. Before this, all these steps had to be done manually, potentially limiting the number of LOMs that could be studied as well as their size. This algorithm was initially developed for 2D RBC (the problem modeled by the Lorenz LOM), and from it came the important result that all physically sound LOMs for this problem that had appeared in recent publications can be converted to G-models, while those lacking such presentation are not energy-conserving (as seen in Gluhovsky, A. and K. Grady, 2016: Effective low-order models for atmospheric dynamics and time-series analysis. *Chaos*, **26**, 023119). The algorithm can also be used to determine if a G-model has a Hamiltonian structure, an important feature to have as the original PDEs are also Hamiltonian.

I have then gone on to adopt this algorithm to study other related problems of atmospheric dynamics. For example, a topic that has received much attention in research on atmospheric convection is how buoyancy and shear interact to form convective patterns of either 2D rolls or 3D cells. Using my algorithm, I derived a G-model for *3D RBC* with shear that incorporated both buoyancy and shear as control parameters. By solving this model for various sets of parameters and plotting the resulting vertical velocity, I was able to visualize what parameter combinations led to either 2D or 3D patterns and thus was able to get at least a basic picture of how buoyancy and shear interact to determine the flow pattern. In general the results from the model were also comparable to observations. The results are presented in a paper, Grady, K. and A. Gluhovsky: Exploring atmospheric convection with physically sound nonlinear low-order models, currently under review for the journal *Communications in Nonlinear Science and Numerical Simulations*. I have also developed G-models for the problem of convection driven by internal heating (the results in a paper under preparation), which is similar to RBC but different in that RBC is driven by heating at the fluid boundary.

In the future I would like to continue this work in using LOMs and in particular G-models to study mathematically the problems of atmospheric and fluid dynamics. There is still much work to be done in the problems that I have addressed so far. In particular, I would like to use the algorithm that I have developed to study even more LOMs of 3D RBC to determine those that result in G-models. Additionally, the theory on developing G-models for the problem of convection driven by internal heating is far from complete and still has several problems that need to be worked out. I would also be interested in collaborating with other researchers to study how a similar algorithm using LOMs could be developed to research the problems of fluid dynamics that they are interested in, or helping to write other programs to study the math and statistics of such problems. I enjoy studying anything that combines my two great academic interests: doing math and studying phenomena of atmospheric and fluid dynamics. LOMs have been a great way to do that, and I would be open to other techniques that allow me to study both as well.